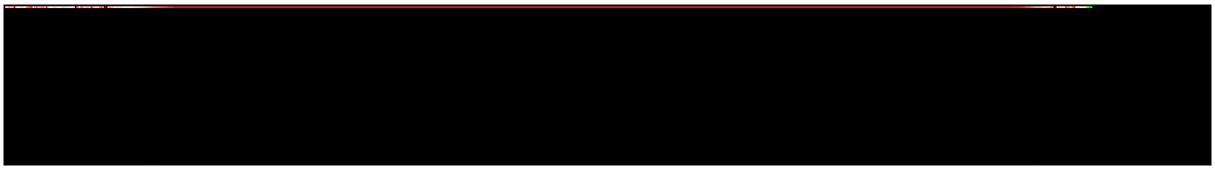

Dynamics in hybrid complex systems of switches and oscillators

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Dynamics in hybrid complex systems of switches and oscillators

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Although extensive theoretical progress has been made in understanding collective behavior in large systems containing a single type of component (such as a switch[1] or oscillator[2]), there has been less development for diverse systems containing more than one type of component. However, many complex systems are composed of various types of units.[3–9] For example, the system-wide dynamics of the yeast cell cycle may be modeled as a system of coupled switches and oscillators.[8,9] Extending the numerical work of Ref. 9, we study interconnected Hopfield switches[10] and Kuramoto oscillators[11] with positive feedback. We find three steady state solutions that may coexist: (i) the Incoherent-Off (I-Off) state in which the oscillators are incoherent and all switches are permanently off, (ii) the Synchronized-On (S-On) state in which the oscillators synchronize and all switches remain on, and (iii) the Synchronized-Periodic (S-P) state in which the oscillators synchronize and the switches periodically turn on and off. Numerical experiments confirm our predictions for these steady state solutions and the transitions between them. Our model demonstrates how the interplay between different units can result in rich dynamics.

I. INTRODUCTION

II. MODEL

$n = 1, \ldots, N$

$$\dot{\theta}_n = \omega_n + \frac{k}{N} \sum_{l=1}^{N} (\theta_l - \theta_n), \qquad (1)$$

$\theta_n(t)$ ... n ... t, ω_n ... n, ... $\Omega(\omega)$, $k(t)$ ... (1), ... M ...

$m = 1, \ldots, M$ ... x_m

$$\dot{x}_m = -x_m - \eta + \frac{K^x}{M} \sum_{l=1}^{M} \tilde{x}_l, \qquad ( )$$

K^x ... $\tilde{x}_m$ ... m ... $\{x_m\}$ ... $\{\tilde{x}_m\}$ ... m,

$\tilde{x}_m = 1$ ($\tilde{x}_m = 0$) f $x_m > 0$ ($x_m \leq 0$) ... 1 ... η ... ( ) ... η ... m ... k,

$$\tau \dot{k} = -k + \frac{K}{M} \ldots$$

$\theta_l \in [0, \pi]$ 0 f $\theta_l \notin [0, \pi]$. , $(\theta_l - \beta_m)$,

.[11] S , . ( ) ( ) k,

f f. , f

. ( ) ( )

f. , f , f

0 f . , f . , ff , , .

A. Parameter choices

f . ( ) ( ) $\Omega(\omega)$ B(β) K, K^x, K^θ, τ, η. f , τ, K > 0. , f . (1) k > 0 $K_0 >$

$\eta > 0$ $N \to \infty$. I 0 0 , 0
0 f 0 0
$\theta_n(t) = \omega_n t + \theta_n(0)$.
I . ( ), 0 0 0 0
0 0 0 0 , 0 0 $N = M = 1000$
0 0 0 0 $k(0) =$,
$\{\theta_n\}$ 0 0 $r_\theta(0) \approx 0$. ,
$\{x_m\}$ 0 0 $r_x(0) \approx 0$ 0 0 $\{x_m(0)\}$ 0 0 -1.
0 , r_x 00 f 0 t, x_m
0 $-\eta$, 0 r_θ k 0 0. . ( ),
0 0 0 f k

C. The synchronized-periodic state

m

$(\tilde{x}_m = 1)$ ff $(\tilde{x}_m = 0)$.

N M

, $\tau \gg \{1, \omega_0^{-1}\}$.

f f

, f . ., . ( ) ( )

k ,

f ,

. ., . ( ) f

. f

τ S . I .

f S-

f $B(\beta)$. f , ,

(1) f

f , $B(\beta) = (\pi)^{-1}$, ( )

, $\beta_m = \beta$ f m. I S . I 1,

f

f

.

1. Unifo ml di ib ed ha e lag

f

$\{\beta_m\}$ f

$[-\pi, \pi]$ . . $B(\beta) = (\pi)^{-1}$ f $\beta \in [-\pi, \pi]$ -

), f

.

S- , ,

$\tau \gg \{1, \omega_0^{-1}\}$, , $N, M \to \infty$.

f S . III , f -

r_x r_θ

r_x . ( )

f $k = Kr_x$. f r_θ

. ( ) $k = Kr_x$ ( ) / 1 f 1 0 . 0 0 0 1 . 0 0 . (.) - . ( )

$\{\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}\}$

I.

I . ( ), (K^x, η)

f , η,

- , K^x.

. ( ) -

. K^x ( )

K^x ( ) η^*.

η , $r_x^{(s)}$ S- .

I . ( ), η^*

f f - K^θ,

$F = 0$, $dF/dr_x = 0$, d $F/dr_x = 0$ f $(r_x$

I . ( ), f -

f - -

, K^x, , η. f

$\eta > \eta^*$ (K^x K^x), -

$\langle r_x \rangle$ S- .

, , η K^x -

. ( ) ( ).

I , f -

S- ff f f -

(. ., r_x

),

. ., . ( ) . ( ) .

I , S- f

f -

f {x_m} (. ., η, K^x,

K^θ) . ( ).

IV. NUMERICAL INVESTIGATIONS

, f

,

f . I S . I

$\sigma_\beta \to 0$. 0 0 0 f B(β) , 0 0
S- 0 0 0 0 0 0 -
0 0 0 0 0 . , f
σ_β f ∞ 0 0, 0 0 0 0 $r_x(t)$
0 0 $\langle r_x \rangle$ 0 0 f 0 -
0 0 f $\sigma_\beta = \infty$ (S . III 1) 0 0 0 -
0 f $\sigma_\beta = 0$ (S . III).
0 0 0 f f 0
0 $r_x(t)$ 0 0 . ( )
0 0 - $\langle r_x \rangle$ f 0 f K^x f $\sigma_\beta \in \{0, 1, 10\}$
. ( ) . I 0 , 00 0 f 0
0 $K^\theta = 10$, K = , K^x

- f τ 0 0 . , 0

f $\langle r_x \rangle$ ( ) f 0 S- 0 0

K^x f f τ. 0 f 0 -

S . III ( ) -

0 0 0 f $\tau = 10$, τ

, 0 $\langle r_x \rangle$ 0 f0 0 0

0 0, 0 0 . f ,

f τ 0 0 (. ., S- 0 0 f

$\tau = 0$.

APPENDIX: THE S-P STATE FOR IDENTICAL PHASE LAGS

I θ , f θ S- θ θ f θ , . ( ) ( ), f θ θ - θ $\beta_m = \beta$ f m. S . III , f θ $N, M \to \infty$

. , - , . , . . , ... 641, (00).
. . . . , S 7, (01).
[10] . . , . . . S . .S. . 79, (1).
[11] . , Chemical Oscillations, Waves, and Turbulence (S , 1).
[1] S. , . , . , . , , . . 424, 1 (00), . . . , ... , The Structure and Dynamics of Networks (, 00).
[1] S. . S et al., 438, (00), . . . , 19, 01 1 (00).
[1] S. . S. . S , 19, 01 1 (00).
[1] . et al., . . 68, 1 1 (1), . . S. , ., . . 72, 00 (1).
[1] . . . 63, (1).
[1] . et al., . S . 19, (00), . . et al., . . . S . .S. . 109, (01).
[1] . et al., ( ) 403, (000), . . 61, (000).
[1] S. et al., S 302, 1 0 (00).
[0] . et al., 14, 0 (00), . et al., . S . 121, (00).
[1] . . . . , S 280, (1).
. , . . . , . . 81, 0 1 (010).
. S. S , . , . . , (01).
. S . , 21, 0 1 (011), S. .
S f , . . 86, 0 1 (01).