P S S [1,2,*] A K [3] J G. R [2]

[1] 43007

[2] 8030

[3] 02115,

(R 26 M 2014; 14 M 2014)

(SDA) P. S. S , A. K , J. G. R , P . R . L . , 108103 (2012) , SDA (C T) . I C T SDA , SDA , . I SDA , . T . A . F . I

DOI: 10.1103/P R E.89.052707 PACS ( ): 87.19.H , 05.45.– , 89.75.–

E 300 000 S [1 3]. F [4 6], [3,7 21]. A (m) (C 2+ i). T (APD) ( ) (C). A . I 2 1 I (SDA) [16], P

SDA . S SDA SDA [10 15,17 21].

T SDA [1,22 25], . . ., . F L, V_m

$$\partial_t V_m = D_V \partial_x^2 V_m - I \quad / C_m, \tag{1}$$

D_V ; I ; C_m ; $x = 0$. V_m . A [26,27] APD . T APD

$$A_{n+1} = f(D_n), \tag{2}$$

---

[*] @ .

A_{n+1} D_n APD (DI) $n+1$ n, . A , (C) , DI, v (D). C $T_n = A_n + D_n$ (n n

11 . I

C T

APD

A

C T

. F

SDA

C - -V_m

T

APD C T

C T

a c,

n

$C_n = C^* + (-1)$

(13) E . (5) (6). T

:

$$c_{n+1}(x) = -rc_n(x) + c_n^3(x) - \alpha a_n(x) + \frac{\alpha}{} \int_0^x e^{(x'-x)/} \; a_n(x')dx', \qquad (19)$$

$$a_{n+1}(x) = \int_0^L G(x,x')\Big[-\beta a_n(x')$$

$$+ \;\beta$$

Eq. (19) [illegible] (20) [illegible]
Eq. (4) [22,23] [illegible]

[illegible]

$$c'(x) = \frac{-\alpha \quad a'(x) - (r-1)c(x) + c^3(x)}{r - 1 - 3c^2(x)}. \tag{33}$$

T , $3c^2(x) = r - 1$ E . (33) $c'(x)$ $c(x)$. T , $c_- = \pm\sqrt{(r-1)/3}$. T

$$(r-1)c(x) - c^3(x) = A(x), \tag{34}$$

$A(x) = -\alpha a(x) + \frac{\alpha}{\ } \int_0^x e^{(x'-x)/\ } a(x')dx'$. S $a(x)$ G , $A(x)$ $c(x)$. T c_+ E . (34) $x = x_0$, $A(x_0) = (r-1)c_- + c_-^3 = \pm 2(r-1)^{3/2}/3\sqrt{3}$, $c_+ = \mp 2\sqrt{(r-1)/3}$. F , $|c_+ - c_-| = \sqrt{3(r-1)}$. I F . 7 $|c_+ - c_-|$

. A

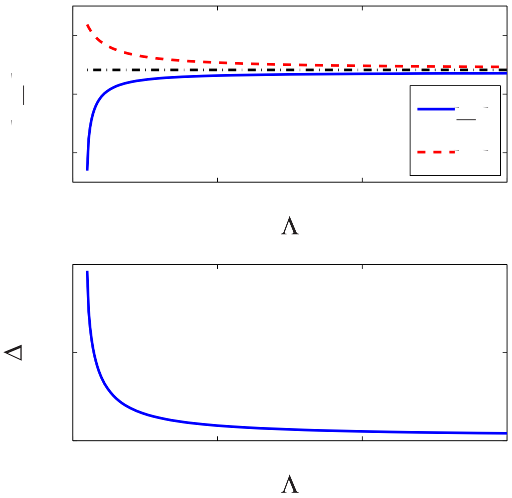


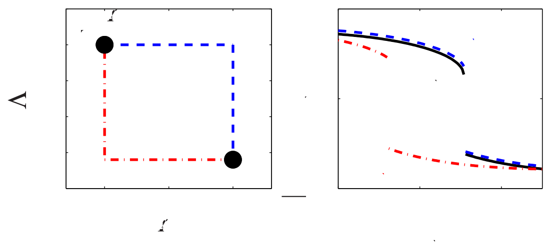
FIG. 9. (C) ( ) J , , $|c_-|$ $|c_+|$ ( ) $r = 1.2$ $= 10,$. O $\alpha,\gamma = \sqrt{0.3}, \beta = 0, \xi = 1,$ $w = 0$ $L = 30$ $x = 0.005.$

$x = 0.005,$ $\alpha,\gamma = \sqrt{0.3}, \beta = 0, \xi = 1,$ $w = 0.$ c_- c_+ . I , E . (19) (20) $w = 0$ $|c_-|$ $|c_+|$ $\sqrt{r-1}$ $\to \infty,$. T

A C. F . 9( ) , . F , , , . F , G , $w > 0,$, c_- , $c_+,$.

N , , r . I , : , , c_- c_+ $x = 0.$ F , r ( ) , , . I , , .

I F . 10 , x_1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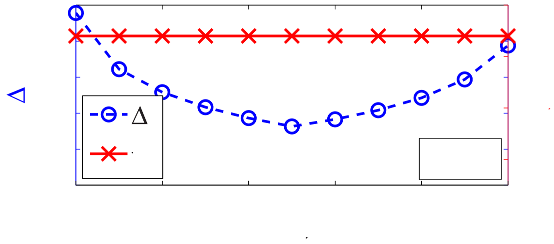
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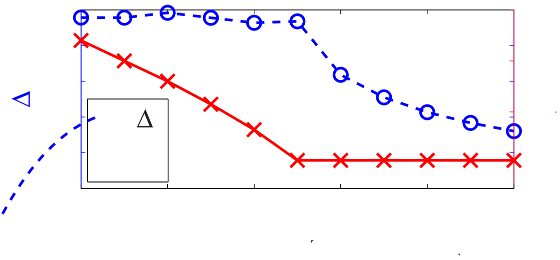
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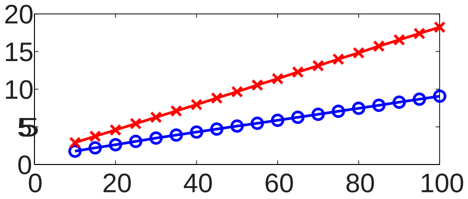
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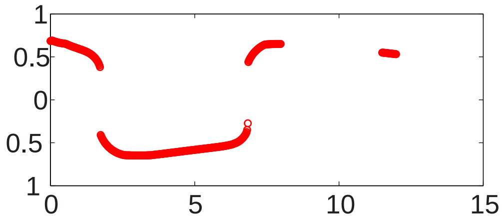
[illegible]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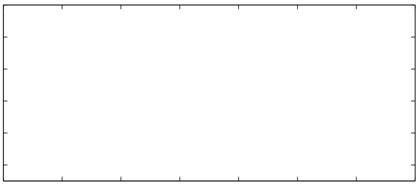
15

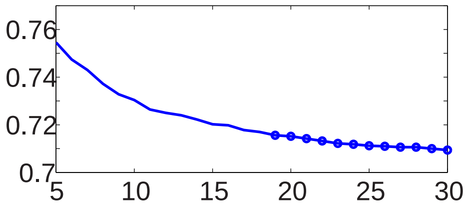
it from 340 ms back to 330 ms. The pacing protocol here is to simulate the cable for 12 000 beats to achieve steady state and then change BCL by BCL = 0.1 ms every 300 beats. In Fig. 20(a) we plot the profile of the amplitude of calcium alternans $c(x)$ at BCL = 330 ms, 335 ms, and 340 ms in red circles, green crosses, and blue triangles, respectively, as we first increase BCL. Note that the node locations move towards the pacing site at $x = 0$ during this process, as predicted by our reduced model, in a similar fashion to changing (see Fig. 19). Furthermore, due to the fixed finite size of the cable, and additional node forms, as it did when changed. In Fig. 20(b) we plot the profile of the amplitude of calcium alternans $c(x)$ as we now decrease BCL, plotting profiles at BCL = 340 ms, 335 ms, and 330 ms in blue triangles, green crosses, and red circles. Importantly, we note that as BCL is restored to 330 ms the node locations remain pinned in their locations close to the pacing site. We again highlight the pinning phenomenon by plotting in Fig. 20(c) the second node location x_3, and versus the beat number in blue circles and dashed red, respectively. Just as in the previous simulation where was modified, we see that the node first moves towards the pacing site as the BCL is initially increased but remains pinned as we restore the BCL to its initial value.

This confirms that unidirectional pinning can be achieved in detailed ionic models by changing only the pacing frequency. However, these results need to be interpreted carefully. In particular, it is well known that a change in BCL results in a change in CV restitution as follows [22,23]: A decrease (increase) in BCL yields a steeper (shallower) CV via decreasing (increasing) DI. However, a change in BCL can also affect change in the degree of calcium instability: A decrease (increase) in BCL allows the calcium dynamics less (more) time to equilibrate between beats, yielding a larger (smaller) degree of instability. Thus, changing the pacing rate yields competing effects from CV restitution and the degree of instability. Here we find that the change in CV restitution is small in comparison to the change in instability, which is dominant. Thus, node movement is induced by decreasing the degree of instability (as predicted by the reduced model and illustrated in Fig. 14), i.e., by increasing BCL. In principle, however, if the change in CV restitution dominates the change in instability, we expect that node movement towards the pacing site will be induced by decreasing BCL.

D. Jump points and asymmetry

of instability. Here we find that the be induced by decreasing BCL.




0.76
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