

Predicting Criticality and Dynamic Range in Complex Networks: Effects of Topology

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The evolution of a complex network is often modeled as a process where nodes are added to an existing network. Here we consider a generalization of this process where nodes are added to a network and each new node is connected to a fixed number of existing nodes. We study the evolution of the network and show that the network exhibits a phase transition at a critical value of the number of nodes. We show that the network is critical at this point and that the dynamic range of the network is maximized at this point. We also show that the network is self-organized to a critical state. Our results have implications for the study of complex networks and for the design of networks.

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Networks are ubiquitous in nature and society [1,2] and have been studied extensively in recent years [3]. One of the most interesting aspects of networks is their ability to exhibit phase transitions. In particular, networks can exhibit a phase transition from a state of low connectivity to a state of high connectivity. This transition is often characterized by a critical point where the network becomes self-organized to a critical state. In this paper, we study the evolution of a complex network and show that the network exhibits a phase transition at a critical value of the number of nodes. We show that the network is critical at this point and that the dynamic range of the network is maximized at this point. We also show that the network is self-organized to a critical state. Our results have implications for the study of complex networks and for the design of networks.

Consider a network of n nodes, where each node i is connected to a fixed number of other nodes, k_i . The network is defined by the adjacency matrix A , where $A_{ij} = 1$ if nodes i and j are connected, and $A_{ij} = 0$ otherwise. The degree of a node i is given by $k_i = \sum_j A_{ij}$. The average degree of the network is given by $\langle k \rangle = \frac{1}{n} \sum_i k_i$. The network is said to be critical if the average degree is equal to the average squared degree, $\langle k \rangle = \langle k^2 \rangle$. This condition is satisfied by a wide class of networks, including random networks, scale-free networks, and small-world networks. In this paper, we study the evolution of a network where nodes are added to the network one at a time, and each new node is connected to a fixed number of existing nodes, k . We show that the network exhibits a phase transition at a critical value of the number of nodes, n_c . For $n < n_c$, the network is in a state of low connectivity, where the average degree is less than the average squared degree. For $n > n_c$, the network is in a state of high connectivity, where the average degree is greater than the average squared degree. At the critical point, $n = n_c$, the network is self-organized to a critical state, where the average degree is equal to the average squared degree. We show that the dynamic range of the network is maximized at this point. The dynamic range is defined as the range of values of a variable that can be observed in the network. We show that the dynamic range is maximized when the network is critical. Our results have implications for the study of complex networks and for the design of networks.

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