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Ensemble-based estimates of eigenvector error for empirical covariance matrices

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1. Introduction

4 r ro r of 4o r 4 m r 4 r 4 r o 4 m m 4 , ro
4 (, 1991 r o , 2003 ., 2009 o & o , 2012) ro 4or r-

o . or m , o o from (1.5) 4 g g o

E[ ] +

Assumption 2.2 m o ro r o (,) of f r g g , + λ λ 1 , ro 4 g λ g fo o g g r g r m for

$$(\;,\;) \quad \frac{3^7 [\; \rho(\lambda\;)]^5}{32\pi^3} [\; ,\; \langle$$

2.3 [illegible] 2 [illegible]

o (2.2) g [illegible] m o 4 m for [illegible]

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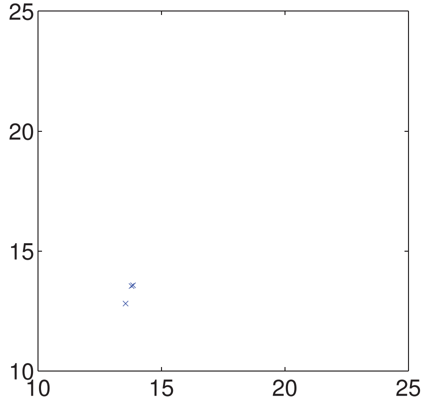
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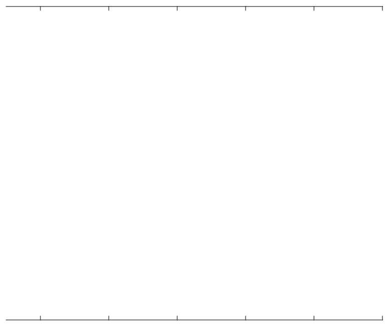

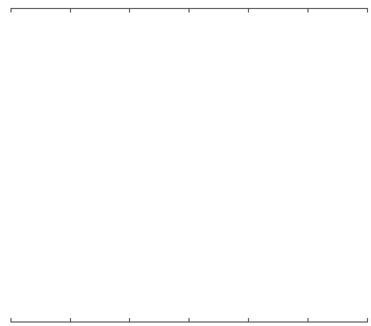

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A. Derivation of main result 1

A. , ro m (1.1) m of r - g o r g g . B o g o, o o k o g of , m g r o of g . g g mm o o o r o

$$+ \quad , \tag{A.1}$$

$$+ \sum_{+1}^{1} \frac{\lambda \lambda}{(\lambda \quad \lambda)^2}, \tag{A.2}$$

$$+ \sum_{+ 1} \frac{\lambda \lambda}{(\lambda \quad \lambda)^2}. \tag{A.3}$$

r m r 4 r m o 4 r - g o r m om o r . T k g o 44o , o r 4 g r r

$$+ \frac{\lambda \lambda_{1}}{(\lambda \quad \lambda_{1})^2} \quad \sum_{+1}^{2} \frac{\lambda \lambda}{(\lambda \quad \lambda)^2}, \tag{A.4}$$

$$+ \frac{\lambda \lambda_{1}}{(\lambda \quad \lambda_{1})^2} \quad \sum_{+ 2} \frac{\lambda \lambda}{(\lambda \quad \lambda)^2}. \tag{A.5}$$

rg o r of (A.4) (A.5) r 4o r g r - g o r m mm o . r 4 r, o ro m o r o o r g g 0 , , (A m o 2.2 4o rg 4 , 4 o , r + (1/)), m o 4 m

$$\frac{\lambda \lambda_{1}}{(\lambda \quad \lambda_{1})^2} \qquad \frac{\lambda^2}{(\quad)^2}. \tag{A.9}$$

o r or o o mm o , ro m g m g , , 4 r $\rho(\lambda)$ of orm m r 4 4o g m r of g . or r 4 , 4o r 4 of - mm r 4 4o r 4 m r 4 , g g λ, for 1, . . . , . for 4 m r m r 4 4 r

$$\rho(\lambda) + \quad ^{1} \sum \delta(\lambda \,), \tag{A.10}$$

r $\delta(\lambda)$ 4 fi 4 o λ R. m 4o r 4 m r 4 r r from m 4 4 $\rho(\lambda)$ k 4o rg , m g

$$\int \rho(\lambda)(\lambda) \;\lambda \qquad \int \rho(\lambda)(\lambda) \;\lambda \tag{A.11}$$

for 4o o fi 4 o (λ). m $\rho(\lambda)$ 4o , o , 4o m 4 or (o (ρ)) ff r o (ρ). or o o 4o 4 , m $(\rho) + (\alpha, \beta)$ for om α, β R, o g o r 4 m of gr o (A.11) (α, β). o r, o r 4 o o of 4 r . g r r g mm o (A.4) (A.5) gr o of fi 4 o

$$(\lambda) + \frac{\lambda \lambda}{(\lambda \quad \lambda)^2} \tag{A.12}$$

ro m r $\rho(\lambda)$ g (A.10),

$$\frac{1}{\quad} \sum_{+1}^{2} \frac{\lambda \lambda}{(\lambda \quad \lambda)^2} + \int_{\alpha}^{\lambda \; 1} \rho(\lambda)(\lambda) \;\lambda, \tag{A.13}$$

$$\frac{1}{\quad} \sum_{+ \; 2\lambda}$$

o r o r g of (A.15) 4 ε. g o $\rho(\lambda)$ (λ) r ff r for λ (ρ) ∥ λ, m m gr o r , r g m r or om or r , o o

$$\int_{\alpha}^{\lambda \quad \varepsilon} (\lambda)\rho(\lambda) \quad \lambda \; + \; \lambda \frac{(\lambda \quad \varepsilon)\rho(\lambda \quad \varepsilon)}{\varepsilon} \quad \lambda \int_{\alpha}^{\lambda \quad \varepsilon} \frac{\rho(\lambda) \quad \lambda\rho\,(\lambda)}{\lambda \quad \lambda} \quad \lambda. \tag{A.16}$$

r m r g of (A.16) ε 0 m o 4 m

$$\lambda \frac{(\lambda \quad \varepsilon)\rho(\lambda \quad \varepsilon)}{\varepsilon} \qquad \frac{\lambda^2\rho(\lambda)}{\varepsilon}. \tag{A.17}$$

4o m o r g of (A.16) o

$$\left|\lambda \int_{\alpha}^{\lambda \quad \varepsilon} \frac{[\rho(\lambda) \quad \lambda\rho\,(\lambda)]}{\lambda \quad \lambda} \quad \lambda\right| \leqslant \lambda \left[\quad_{(\alpha,\lambda \quad \varepsilon]} \rho(\lambda) \quad \lambda\rho\,(\lambda) \right] \int_{\alpha}^{\lambda \quad \varepsilon} \frac{1}{\lambda \quad \lambda} \quad \lambda$$

$$+ \lambda \left[\quad_{(\alpha,\lambda \quad \varepsilon]} \rho(\lambda) \quad \lambda\rho\,(\lambda) \right] \quad \left(\frac{\lambda \quad \alpha}{\varepsilon}\right). \tag{A.18}$$

fo o 4o m r g of (A.16) 4 g ($(1/\varepsilon)$) om m ε 0 r m, 4 $(1/\varepsilon)$. 4om (A.17) (A.18) o o ε 0 m o 4 m

$$\int_{\alpha}^{\lambda \quad \varepsilon} (\lambda)\rho(\lambda) \quad \lambda \qquad \frac{\lambda^2\rho(\lambda)}{\varepsilon}. \tag{A.19}$$

o 4 r $\rho\,(\lambda)$ o , r g for r o r gr o f of (A.18) o o om : o 4o g λ, r $\rho\,(\lambda)$ o 4o om g r m λ ε. r gr 4o rg o ro o (A.11) r 4o o g (A.18), m g gr m (A.16) ($(1/\varepsilon)$), ro $\rho(\lambda)$ ff r m g o r oo 4o g λ . m g o r for r g of (A.13) 4o r g fo o g :

$$\int_{\alpha}^{\lambda \quad 1} (\lambda)\rho\,(\lambda) \quad \lambda + \int_{\alpha}^{\lambda} (\lambda)\rho(\lambda) \quad \lambda \quad \int_{\alpha}^{\lambda} (\lambda)\left[\rho\,(\lambda) \quad \rho(\lambda)\right] \quad \lambda$$

ı \m\m r , 4o\m (A.21), (A.22) (A.17

To o m for r o of (), ff r (B.6) r 4 o oo

$$(\,) + \frac{\partial}{\partial} \int_{0()} \int_{(,)} (\;,\;) \qquad \text{(B.7)}$$

$$+ \frac{\partial\, ^0}{\partial}() \int_{(,\,^0())} (\;,\, ^0()) \quad \int_{0()} \frac{\partial}{\partial} \left[\int_{(,)} (\;,\;) \right]$$

$$+ \int_{0()} ((,),) \frac{\partial (,)}{\partial} .$$

o o r o, r m 4o 4 (,) , m $^0()$ (,) o .

C. Derivation of main result 3

r 44or g o (), g (B.7), r 4 o m o 4 r o for () m , . m g (2.2), o =(,) rg /or r m, rg m o 4 4o r o 4o r o of m ro or o o 2 2,

$$\frac{\lambda^2}{(\;)^2} \quad \frac{\lambda^2}{(\;)^2}. \qquad (.1)$$

4 ,

$$^0() + \frac{\lambda}{}, \qquad (.2)$$

$$(,) + \frac{\lambda}{[(\;)^2 \quad \lambda^2]^{1/2}}, \qquad (.3)$$

$$\frac{\partial}{\partial}\big((,)\big) + \frac{\lambda(\;)^3}{2\,[(\;)^2 \quad \lambda^2]^{3/2}} + \frac{1}{2\lambda^2}[(,)]^3. \qquad (.4)$$

g o (B.7) ro g rg m for , . . . (,) / ,

$$() + \int_{\lambda^2/} \left(\frac{3^7 [\rho(\lambda)]^5}{32\pi^3} (\;,\;) \quad \frac{[3\rho(\lambda)]^2}{4\pi} [(\;)^2 (\;)^2] \right) \left(\frac{(\;)^3}{2\lambda^2} \right)$$

$$+ \frac{3^7 [\rho(\lambda)]^5}{32\pi^3} \frac{1}{2\lambda^2} \int_{\lambda^2/} \left(\left((\;)^5 \quad (\;)^2 (\;)^4 \right) \quad \frac{[3\rho(\lambda)]^2}{4\pi} [(\;)^2 (\;)^2] \right) .$$

4 g of r $+ (\;)^2 \quad \lambda^2$ r form o

$$() + \frac{3^7 [\rho(\lambda)]^5}{32\pi^3} \frac{\lambda^2}{4} \;^{7/2}\, (), \qquad (.5)$$

o , 4 g for gr $_1(\cdot)$, $_2(\cdot)$ $_3(\cdot)$. B g g (.4), r o for r g (0, $_1(\cdot)$]

$$(\lambda^2)^{5/2}(^{1/2} \lambda) \leqslant (_1 \lambda^2)^{5/2}(_1^{1/2} \lambda). \quad (.7)$$

fo o

$$\left(1 \frac{\lambda^2}{}\right)$$ fo o

$$\left(^{/2} \right.$$

r 4o > 0 $\lambda^2/$ > 0 o

$$\varphi(\ ) + \frac{[3\ \rho(\lambda)]^2}{4\pi}(\ \ \lambda^2)\left[1\ \ (\lambda^2/\ )\ \ (\lambda^2/\ )^{1/2}\right]$$

$$\geqslant \frac{[3\ \rho(\lambda)]^2}{4\pi}\ . \qquad (\ .17)$$

g of r $+ \frac{[3\ \rho(\lambda)]^2}{4\pi}$ o o

$$_3(\ ) \leqslant 8\left(\frac{[3\ \rho(\lambda)]^2}{4\pi}\right)^{3/2}\int_{\frac{[3\ \rho(\lambda)]^2}{4\pi}\ _2(\ )/}\ ^{1/2}\ . \qquad (\ .18)$$

rom (.3), o r m of gr o 4o rg $\frac{[3\ \rho(\lambda)]^2}{4\pi}\ _2(\ )/$ 1 gr (.18) 4o rg o 4o . r for , $_3(\ )$ o m 4 g $^{3/2}$. o o $_2(\ )$ 4 g $(\ ^{3/2})$ (o o o o r m , o r m 4 $^{3/2}$). o 4 of r o of $_1$ $_2$, g $\varphi(\ )$ r m m m $+ \lambda^2$ $(\ _1,\ _2)$,

$$\varphi(\lambda^2)/\ \leqslant \varphi(\ )/\ \leqslant 1 \qquad (\ .19)$$

for $(\ _1,\ _2)$. g o (.6), $_2(\ )$

$$(\ )\ ^1 \leqslant\ _2(\ ) \leqslant\ (\ )\ ^{\varphi(\lambda^2)/},\qquad (\ .20)$$

r .

$$(\ )\int_{_1(\ )}^{_2(\ )}\left(1\ \ \frac{\lambda^2}{}\right)^{5/2}(\ ^{1/2}\ \ \lambda)\ . \qquad (\ .21)$$

g m o 4 ro m o for $_1(\ )$ $_2(\ )$ g (.2) (.3), gr (.21) g of r m 4 (g r [( ), ,1]' 4o m) o o m o 4 o r,

$$(\ )\ \frac{2^4\pi^{3/2}}{3^4[\ \rho(\lambda)]^3}\ ^{3/2}. \qquad (\ .22)$$

r mor , 4o m $\varphi(\lambda^2)/$ 0 (.20) o o m o 4

$$(\ )\ ^1 \leqslant\ _2(\ ) \leqslant\ (\ ). \qquad (\ .23)$$

4o m (.23) (.15) (.18) o o rg 4 g $(\ ) + \left(\frac{^{3/2}}{^3}\right)$.