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physical perturbations.[12,13] Other previous works have investigated the inference of network links from time series of node states assuming some prior knowledge of the form of the network system and using that knowledge in a fitting procedure to determine links.[9,14–17] In addition, some recent papers address network link inference from data via techniques based on delay coordinate embedding,[15] random forest methods,[18] network embedding algorithms,[19] and feature ranking.[20] In this paper, we introduce a technique that makes the use of an ML training process in performing predictive and interpretive tasks and attempts to use it to extract information about causal dependencies. In particular, here, we use a particular type of machine learning (ML) called reservoir computing, an efficient method of

not possible, we note the case where the attractor is a fixed point (a zero-dimensional attractor). Here, the measured available information is the M numbers that are the coordinates of the fixed point, and this information is clearly insufficient for determining STCD. As another problematic example, we note that in certain cases, one is interested in a dynamical system that is a connected network of identical dynamical subsystems and that such a network system can exhibit exact synchronization of its component subsystems[26] (including cases where the subsystem orbits are chaotic). In the case where such a synchronized state is stable, observations of the individual subsystems are indistinguishable, and it is then impossible, in principle, for one to infer causal relationships between state variables belonging to different subsystems. More generally, in addition to the above fixed point and synchronization examples, we note that the dimension of the tangent space at a given point z on the attractor is, at

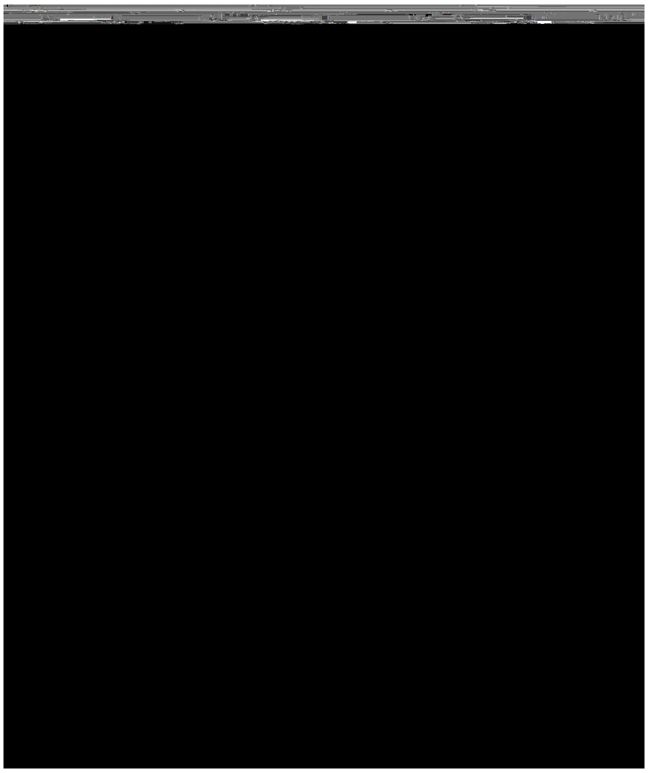

FIG. 3. The effect of noise on STCD inference. Panels (a)–(c) show the effect of increasing the dynamical noise variance $^{2}_{\mathrm{Dyn}}$ to greatly enhance the effectiveness of link identification even at the rather low noise level of $^{2}_{\mathrm{Dyn}} = 10\ ^{6}$. In contrast, as shown in panels (d)–(f), starting with the situation (c) and increasing the observational noise variance $^{2}_{\mathrm{Obs}}$ degrades link identification. $L = 200$, $h = 0$ for all the subfigures here.

The state space dimension of this system is $M = 3N$. The coupling of the N nodes is taken to be only from the y variable of one node to the x variable of another node with coupling constant c, and $a_{kl}^{(x,y)}$ is either 1 or 0 depending on whether or not there is a link from l to k. The adjacency matrix $a_{kl}^{(x,y)}$ of our Lorenz network (not to be confused with the adjacency matrix A of the reservoir) is constructed by placing directed links between L distinct randomly chosen node pairs. For each node k, h_k is randomly chosen in the interval $[\ h, +h]$, and we call h the heterogeneity parameter. Independent white noise terms of equal variance $^{2}_{\mathrm{Dyn}}$ are added to the left-hand sides of the equations for dx/dt, dy/dt, and dz/dt, where, for example, $\langle n_{kx}(t) n_{k x}(t) \rangle = 2\ _{kk}\ (t\ \ t)$. For $\ = c = h = 0$, each node obeys the classical chaotic Lorenz equation with the parameter values originally studied by Lorenz.[39] Furthermore, denoting the right-hand side of Eq. (5) by F

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